Colloquium on General and Set-Theoretic Topology

dedicated to the 60th birthday of **István Juhász**

organized by the János Bolyai Mathematical Society in cooperation with the Paul Erdős Summer Research Center of Mathematics and the Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences

Abstracts

Budapest, August 8–13, 2003

Supported by the "Mathematics in Information Society" project (in the framework of the European Community's "Confirming the International Role of Community Research" programme)

Contents

Cardinal invariants in topological spaces with some algebraic structures

A. V. Arhangel'skii

(Ohio University & University of Moscow)

(arhangel@bing.math.ohiou.edu)

Zero-Divisor Graph of $C(X)$

F. Azarpanah

(Chamran University, Iran)

(azarpanah@ipm.ir)

joint research with M. Motamedi

We denote the zero-divisor graph of $C(X)$ by $\Gamma(C(X))$ and we will associate the ring properties of $C(X)$, the graph properties of $\Gamma(C(X))$ and the topological properties of X. Cycles in $\Gamma(C(X))$ are investigated and an algebraic and a topological characterization is given for the graph $\Gamma(C(X))$ to be triangulated or hypertriangulated. We have shown that the clique number of $\Gamma(C(X))$, the cellularity of X and the Goldie dimension of $C(X)$ coincide. It turns out that the dominating number of $\Gamma(C(X))$ is between the density and the weight of X. Finally we have shown that $\Gamma(C(X))$ is triangulated and the set of centers of $\Gamma(C(X))$ is a dominating set if and only if the set of isolated points of X is dense in X if and only if the Socle of $C(X)$ is an essential ideal.

A new approach to the degree of points

Mátyás Bognár

(Eötvös University Budapest)

(bognar@ludens.elte.hu)

The degree of a point (or the order of a point) is a fundamental concept in mender-Urysohn curve theory (see e.g. C. Kuratowski, Topologie II). It can be defined in arbitrary topological spaces, however sometimes it does not coincide with the visual requirements.

Let N_0 be the set of nonnegative integers.

For a subset M of a topological space (E, τ) let $Comp_\tau(M)$ denote the family of the components of the set M in (E, τ).

The number of elements of a set Q will be denoted by $\underline{n}(Q)$, where $\underline{n}(Q) \in N_0 \cup \{\infty\}$.

Let (E, τ) be a topological space. Let $p \in E$ and let $m \in N_0$. We write $dg_\tau(p)|m$ if to each neighbourhood U of p (i.e. to each $U \in \tau(p)$) there is $V \in \tau(p)$ such that $V \subset U$ and $\underline{n}(Comp_\tau(V \setminus \{p\})) \leq m$.

We say that the degree of p with respect to (E, τ) is infinite and we write $deg_\tau(p) = \infty$ if there is no $m \in N_0$ with $dg_\tau(p)|m$. Otherwise we say that this degree is finite and its definition is

$$deg_\tau(p) = \min\{m \in N_0 : dg_\tau(p)|m\}.$$

Line complexes are practically locally finite graphs without loops. If W is a line complex, τ its natural topology, τ' the topology of $\cup W$ and $A = \{p\}$ is a node of W then $deg_\tau(A) = deg_{\tau'}(p)$, while the usual concept may have results $ord_A W \neq ord_p(\cup W)$.

Every Eberlein compact space Y has a dense G_δ metrizable subspace X with $\dim X \leq \dim Y$

M.G. CHARALAMBOUS

(University of the Aegean, Greece)

(mcha@aegean.gr)

An Eberlein compact space is a compact subspace of $c_0(S)$ for some set S. Here $c_0(S)$ is the subspace of the product of unit intervals $\prod_{s\in S} \mathbb{I}_s$ consisting of the points $x = (x_s)$ such that the set $\{s \in S : x_s > \epsilon\}$ is finite for each positive real number ϵ.
G. Dimov has proved that a Baire subspace Y of $c_0(S)$ contains a dense G_δ subspace X which is metrizable, extending a result previously known for Y Eberlein compact. We prove that $\dim X \leq \dim Y$. This follows from certain results that we establish concerning the dimension of uniform spaces.
It should be noted that for a dense metrizable subspace X of a compact space Y, we do not necessarily have $\dim X \leq \dim Y$. For Roy's example of a metrizable space X with $\dim X = 1$ and $indX = 0$ has a compactification Y with $\dim Y = 0$.
It is also of interest to note that T. Kimura and K. Morishita have recently proved that every metrizable space X has an Eberlein compactification Y with $\dim Y = \dim X$. In fact, one can even show that $IndY = \dim X$.

Generalized open sets in generalized topologies

ÁKOS CSÁSZÁR

(Eötvös University Budapest)

In the topological literature, one finds a series of classes of sets that are generalizations of open sets: semi-open sets, preopen sets, α-open sets, β-open sets. Each of these classes is a generalized topology (GT), i.e it contains $\emptyset$ and the union of an arbitrary subclass.

The purpose of the present talk is a modification of the definition of the above classes in the following sense. Let λ be a GT on a set X. Define, for $A \subset X$,

$$i_\lambda A = \bigcup\{L \in \lambda : L \subset A\}, \quad c_\lambda A = \bigcap\{X - L : L \in \lambda, A \subset X - L\}.$$

Now let us replace, in the definition of semi-open, preopen, α-open, β-open sets, the operation $int(A)$ by i_λ and $cl(A)$ by c_λ; more precisely, let us say that $A \subset X$ is λ-semi-open iff $A \subset c_\lambda i_\lambda A$, λ-preopen iff $A \subset i_\lambda c_\lambda A$, λ-α-open iff $A \subset i_\lambda c_\lambda i_\lambda A$, λ-β-open iff $A \subset c_\lambda i_\lambda c_\lambda A$. Denote these classes by $\sigma(\lambda)$, $\pi(\lambda)$, $\alpha(\lambda)$, $\beta(\lambda)$, respectively. Each of these classes is in GT, so one can speak of the class of $\xi(\eta(\lambda))$ for $\xi, \eta = \sigma, \pi, \alpha, \beta$.

We show that in some cases the class $\xi(\eta(\lambda))$ coincides with one of the classes $\sigma(\lambda)$, $\pi(\lambda)$, $\alpha(\lambda)$, $\beta(\lambda)$, but, in general, one obtains new classes of sets. As an application, we get characterization of the generalized topologies of the form $\sigma(\tau)$ where τ is a topology on X.

Compact spaces of countable tightness

Alan Dow

(University if North Carolina at Charlotte)

(adow@uncc.edu)

Covering properties and preopen sets

Maximilian Ganster

(Graz University of Technology)

(ganster@weyl.math.tu-graz.ac.at)

joint research with M. Sarsak

We consider various covering properties which involve preopen sets in their definitions. Special attention will be paid to the case of metacompactness, - a recent joint research project with Prof. M. Sarsak from Hashemite University, Jordan.

A unified theory of $T_{\frac{1}{2}}$ spaces

D. N. Georgiou

(Department of Mathematics, University of Patras, 265 00 Patras, Greece)

(georgiou@math.upatras.gr)

joint research with M. Caldas, S. Jafari and T. Noiri

In 1970, Levine [4] introduced the notion of $T_{\frac{1}{2}}$ spaces which lie properly between T_1-spaces and T_0-spaces. Dunham [3] obtained the following characterization of $T_{\frac{1}{2}}$-spaces: a topological space (X, τ) is $T_{\frac{1}{2}}$ if and only if each singleton of X is open or closed. Moreover, Arenas et al. [1] showed that a topological space (X, τ) is $T_{\frac{1}{2}}$ if and only if every subset of X is τ-closed. In 1987, semi-$T_{\frac{1}{2}}$ spaces are introduced by Bhattacharyya and Lahiri [3]. Sundaram et al. [5] showed that a topological space (X, τ) is semi-$T_{\frac{1}{2}}$ if and only if each singleton of X is semi-open or semi-closed.

In this paper, we introduce the notions called m-structures which are weaker than topological structures. Using the m-structures, we investigate a unified theory of weak separation axioms containing $T_{\frac{1}{2}}$ spaces and semi-$T_{\frac{1}{2}}$ spaces.

References

[1] F. G. Arenas, J. Dontchev and M. Ganster, On λ-sets and dual of generalized continuity, Questions Answers Gen. Topology, **15**(1997), 3-13.

[2] P. Bhattacharyya and B. K. Lahiri, Semi-generalized closed sets in topology, Indian J. Math. **29**(1987), 375-382.
[3] W. Dunham, $T_{\frac{1}{2}}$-spaces, Kyungpook Math. J., **17**(1977), 161-169.
[4] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo **19**(2)(1970), 89-96.
[5] P. Sundaram, H. Maki and K. Balachandran, semi-generalized continuous maps and semi-$T_{\frac{1}{2}}$ spaces, Bull. Fukuoka Univ. Ed. Part III **40**(1991), 33-40

On left-separated spaces

JÁNOS GERLITS

(Alfréd Rényi Institute of Mathematics)

(gerlits@renyi.hu)

joint research with ISTVÁN JUHÁSZ, LAJOS SOUKUP AND ZOLTÁN SZENTMIKÓSSY

Embeddability of Countable Metric Spaces

WILLIAM D. GILLAM

(Wesleyan University)

(wgillam@wesleyan.edu)

We discuss some general results about countable metric spaces (all of which are, up to homeomorphism, subspaces of the rationals) with an emphasis on scattered spaces (i.e. spaces with no crowded subspaces) and properties related to homeomorphic embeddability. Using invariants for relatively "simple" spaces (those of finite Cantor-Bendixson rank), together with a technical result reducing questions of embeddability to this case, we prove two theorems: If F is a set of countable metric spaces such that no space in F embeds in any other space in F, then F is finite. (Arbitrarily large such finite sets exist.) If G is any non-empty set of countable metric spaces then there is some X in G so that, for any Y in G, if Y embeds in X then X embeds in Y.

Construction of uniform homeomorphisms between spaces equiped with the pointwise topology

RAFAL GORAK

(INSTITUTE OF MATHEMATICS of the Polish Academy of Sciences, Warszawa, Poland)

(gorakrafal@yahoo.com)

In my talk I will present the method of constructing uniform homeomorphisms between real

valued function spaces equipped with the topology of pointwise convergence (denoted usually by $C_p(X)$) invented by Gulko and developed by myself to solve the following problems:

i) I will shortly present my result concerning characterization of spaces X such that $C_p(X)$ and $C_p(\mathbf{I}^n)$ are uniformly homeomorphic ($\mathbf{I}^n$ is the standard n-th cube) .

ii) I will also focus on spaces $C_p([0,\alpha])$ where α is an ordinal and $[0,\alpha] = \{\beta : \beta \leq \alpha\}$ endowed with the standard order topology. I will give the uncomplete classification of such spaces up to (uniform) homeomorphisms and I will state the conjecture what the complete classification is.

Containing spaces and actions of groups.

S.D. Iliadis

(Department of Mathematics, University of Patras, 265 00 Patras,Greece)

(iliadis@math.upatras.gr)

All considered spaces are assumed to be T_0-spaces of weight less than or equal to a given infinite cardinal τ. Let G be an arbitrary topological group. In parallel to the notions of a saturated class of spaces and a saturated class of mappings we introduce the notion of a saturated class of G-spaces. We give the basic properties of these classes, that is, in any such class there exists universal elements and the intersection of saturated classes is also such a class. We prove that for any saturated class P of spaces the class of all G-spaces of P is a saturated class of G-spaces. In particular, we have the following consequence.

The following classes of G-spaces are saturated:

(1) The class of all regular (completely regular) G-spaces.

(2) The class of all regular (completely regular) countable-dimensional G-spaces.

(3) The class of all regular (completely regular) strongly countable- dimensional G-spaces.

(4) The class of all regular (completely regular) locally finite- dimensional G-spaces.

(5) The class of all regular (completely regular) G-spaces of dimension ind less than or equal to an ordinal $\alpha \in \tau^+$ (in particular, α may be an non-negative integer).

Non-degenerate functions in the sense of Bott

Saeid Jafari

(Department of Mathematics and Physics, Roskilde University, Postbox 260, 4000 Roskilde, Denmark)

(sjafari@ruc.dk)

In this talk, we discuss the connectivity of the non empty levels of non-degenerate functions in the sense of Bott on a compact connected manifold.

Resolvability versus maximal resolvability

I. JUHÁSZ

(Alfréd Rényi Institute of Mathematics)

(juhasz@renyi.hu)

joint research with L. SOUKUP AND Z. SZENTMIKLÓSSY

In this joint work with L. Soukup and Z. Szentmiklóssy we prove several new results concerning resolvability. Our main tool is a very general construction method in ZFC that allows us to refine topologies in such a way that in the resulting finer topology a set will be dense in an open set only if some prescribed family of dense sets "forces" this. As a corollary, for every uncountable regular cardinal κ we construct a 0-dimensional T_2 space X with $\Delta(X) = \kappa$ that is λ-resolvable for all $\lambda < \kappa$ but is not κ-resolvable. This solves an old question of Ceder and Pearson.

We also strengthen a recent result of O. Pavlov by proving that for every regular cardinal κ if the space X satisfies $\Delta(X) \geq \kappa$ and X has no discrete subspace of size κ, then X is κ-resolvable.

References

[1] J. Ceder and T. Pearson, *On products of maximally resolvable spaces*, Pacific J. Math. **22** (1967), 31–45.

[2] O. Pavlov, *On resolvability of topological spaces*, Topology and its Applications, 126 (2002), 37–47.

Hechler's theorem for ideals of the reals

MASARU KADA

(Kitami Institute of Technology, Kitami, JAPAN)

(kada@math.cs.kitami-it.ac.jp)

Hechler's theorem, which is a classical result of the theory of forcing, is the following statement: For a partially ordered set Q such that every countable subset has a strict upper bound, there is a forcing notion satisfying ccc such that, in the forcing model, there is a cofinal subset of $(\omega^\omega, \leq^*)$ (the set of all functions from ω to ω ordered by eventually dominating order) which is order-isomorphic to Q with respect to set-inclusion.

We show that statements similar to the above hold for the meager ideal and the null ideal of the reals, ordered by set-inclusion. Both of them are proved using a method of forcing construction similar to the one in Hechler's original proof.

On Kash spaces

O.A.S. Karamzadeh

(Chamran University, Iran)

(karamzadeh@cua.ac.ir)

joint research with A.A. Staji

Email: karamzadeh@cua.ac.ir Coauthors: Title: Abstract: Let c be a cardinal number, then a commutative ring R is said to be a c-Kash-ring if ideals with generating set of cardinality less than c are non-essential and are maximal with respect to this property. A topological space X is called c-Kash if $C(X)$ is a c-Kash ring. We observe that X is an almost P-space if and only if X is $\aleph_0$- Kash. It is also shown that X is $\aleph_1$-Kash if and only if X is a pseudocompact almost P-space. X and βX are shown to be c-Kash for the same c. Some other properties of c-Kash spaces are discussed.

On hyperspace topologies

Ljubiša D.R. Kočinac

(Faculty of Philosophy, University of Niš, 18000 Niš, Yugoslavia)

(kocinac@archimed.filfak.ni.ac.yu)

For a space X consider the set of all closed subsets of X endowed with different (upper) topologies. We discuss duality between X and its hyperspaces considering some properties given in terms of selection principles.

Transitive properties of ideals

Jan Kraszewski

(University of Wroclaw, Poland)

(kraszew@math.uni.wroc.pl)

We consider an invariant σ-ideal of subsets of an infinite Abelian group $(G, +)$. For such an ideal we define the following cardinal coefficients.

$\mathrm{add}_t(\mathcal{J}) = \min\{|\mathcal{A}| : \mathcal{A} \subseteq \mathcal{J} \ \& \ \neg(\exists B \in \mathcal{J})(\forall A \in \mathcal{A})(\exists g \in G)\, A \subseteq B + g\}$,
$\mathrm{add}^*_t(\mathcal{J}) = \min\{|T| : T \subseteq G \ \& \ (\exists A \in \mathcal{J})\, A + T \notin \mathcal{J}\}$,
$\mathrm{cov}_t(\mathcal{J}) = \min\{|T| : T \subseteq G \ \& \ (\exists A \in \mathcal{J})\, A + T = G\}$,
$\mathrm{cof}_t(\mathcal{J}) = \min\{|\mathcal{B}| : \mathcal{B} \subseteq \mathcal{J} \ \& \ \mathcal{B} \textit{ is a transitive base of } \mathcal{J}\}$,

where a family $\mathcal{B} \subseteq \mathcal{J}$ is called a *transitive base* if for each $A \in \mathcal{J}$ there exists $B \in \mathcal{B}$ and $g \in G$ such that $A \subseteq B + g$. First two ones are both called *transitive additivity*. The latter two ones are called *transitive covering number* and *transitive cofinality*, respectively.

We say that an ideal $\mathcal{J}$ is $\kappa - translatable$ if

$$(\forall A \in \mathcal{J})(\exists B \in \mathcal{J})(\forall T \in [G]^{\kappa})(\exists g \in G)\, A + T \subseteq B + g.$$

We define a *translatibility number* of $\mathcal{J}$ as follows

$$\tau(\mathcal{J}) = \min\{\kappa : \mathcal{J} \textit{ is not } \kappa - translatable\}.$$

We introduce also some operations on $\mathcal{J}$.

$s(\mathcal{J}) = \{A \subseteq G : (\forall B \in \mathcal{J})\, A + B \neq G\}$,
$g(\mathcal{J}) = \{A \subseteq G : (\forall B \in \mathcal{J})\, A + B \in \mathcal{J}\}$,

We investigate properties of these operations and cardinal coefficients. In particular, we show what happens for $\mathcal{J} = \mathbb{S}_2$, where $\mathbb{S}_2$ denotes the least nontrivial σ-ideal of subsets of the Cantor space 2^{ω}.

The Complex Stone-Weierstrass Property and Elementary Submodels

KENNETH KUNEN

(University of Wisconsin)

(kunen@math.wisc.edu)

The compact Hausdorff space X has the Complex Stone-Weierstrass Property (CSWP) iff X satisfies the complex version of the Stone-Weierstrass Theorem (that is, every algebra of continuous complex-valued functions on X which separates points and contains the constant functions is dense in $C(X)$).

By results of W. Rudin and others, the CSWP was known to be true of all compact scattered spaces, and false of spaces containing a copy of either the Cantor set or βN. It was not known to be true of any non-scattered space.

By applying elementary submodel techniques, one can show that in fact the CSWP holds for a number of non-scattered compact spaces. For example, a compact LOTS satisfies the CSWP iff it does not contain a copy of the Cantor set.

Some remarks on the existence of Tychonoff-Topologies on space-time-manifolds

OTTO LABACK

Name:
University of Technology Graz Dept. mathematic

There are natural semimetric topologies on space-time manifolds but until now no such completely regular topologies. We try to explain this open problem and will give some remarks on modelling the causality of space-time manifolds.

On lifted closure operators

GÁBOR LUKÁCS

(Department of Mathematics Statistics, York University, 4700 Keele Street Toronto, Ontario, M3J 1P3 Canada)

(lukacs@mathstat.yorku.ca)

For a topological group G, we denote by $\kappa_G : G \to bG$ the *Bohr-compactification of* G, and one sets $G^+ = \kappa_G(G)$; G^+ is a dense subgroup of bG. The *Bohr-closure* of a subset $A \subseteq G$ is the closure of A in the initial topology induced on G by the map κ_G. In other words, $c_b(A) = \kappa_G^{-1}(\overline{\kappa_G(A)}^{G^+})$. A group K is called c_b*-compact* if the projection $\pi_H : K \times H \to H$ maps c_b-closed subgroups of the $K \times H$ onto c_b-closed subgroups of H. We observe the following property of c_b:

Theorem . *A topological group G is c_b-compact if and only if it $G^+ = bG$ (i.e. κ_G is surjective).*

A *closure operator* on $\mathfrak{X}$ ([1]) is a family of maps $c_X : \mathrm{sub}(X) \longrightarrow \mathrm{sub}(X)$, $X \in \mathfrak{X}$, such that: (1) $m \leq c_X(m)$ for all $m \in \mathrm{sub}(X)$ (extensive); (2) $m \leq n$ implies $c_X(m) \leq c_X(n)$ for all $m, n \in \mathrm{sub}(X)$ (monotone); (3) $f(c_X(m)) \leq c_Y(f(m))$ for all $m \in \mathrm{sub}(X)$, for every morphism $f : X \to Y$ (morphisms are continuous).

A morphism $f : X \to Y$ in $\mathfrak{X}$ is *c-preserving* if $c_Y(f(m)) = f(c_X(m))$ for every $m \in \mathrm{sub}(X)$. Following [2], an object $X \in \mathfrak{X}$ is *c-compact* if for every $Y \in \mathfrak{X}$ the projection $\pi_Y : X \times Y \to Y$ is c-preserving.

Notice that the closure operator c_b was obtained as the *lifting* of the Kuratowski closure operator through the Bohr-reflection. It turns out that we can lift any closure operator from a nice enough subcategory. Let $\mathfrak{A}$ be an epireflective subcategory of $\mathfrak{X}$ with reflector R and unit ρ. Given a closure operator c on $\mathfrak{A}$, c can be lifted to a closure operator c^ρ on $\mathfrak{X}$ defined by $c^\rho_X(m) = \rho_X^{-1}(c(\rho_X(m)))$. We show an analogue of the Theorem: under mild conditions, $X \in \mathfrak{X}$ is c^ρ-compact if and only if RX is c-compact.

1. M. M. Clementino, E. Giuli, and W. Tholen. Topology in a category: compactness. *Portugal. Math.*, 53(4):397–433, 1996.

2. Dikran Dikranjan and Eraldo Giuli. Compactness, minimality and closedness with respect to a closure operator. In *Categorical topology and its relation to analysis, algebra and combinatorics (Prague, 1988)*, pages 284–296. World Sci. Publishing, Teaneck, NJ, 1989.

Cardinal sequences of scattered Boolean spaces

JUAN CARLOS MARTÍNEZ

(Facultat de Matemàtiques, Universitat de Barcelona, Gran Via 585, 08007 Barcelona, Spain)

(martinez@mat.ub.es)

The Cantor-Bendixson process for topological spaces is defined as follows. Suppose that X is a topological space. Then, for every ordinal α we define the α*-derivative of* X by: $X^0 = X$; if

$\alpha = \beta + 1$, X^α is the set of accumulation points of X^β; and if α is a limit, $X^\alpha = \bigcap\{X^\beta : \beta < \alpha\}$. Then, X is *scattered*, if $X^\alpha = \emptyset$ for some ordinal α.

The Cantor-Bendixson process permits us to split a scattered space into levels. Suppose that X is a scattered space. We define the *height of* X by $\mathrm{ht}(X) =$ the least ordinal α such that X^α is finite. For $\alpha < \mathrm{ht}(X)$, the *set of points at level* α in X is the set $I_\alpha(X) = X^\alpha \setminus X^{\alpha+1}$. The *cardinal sequence* of X is defined by $\mathrm{CS}(X) = \langle |I_\alpha(X)| : \alpha < \mathrm{ht}(X)\rangle$.

Then, we shall expose a series of results on cardinal sequences of scattered Boolean spaces as well as a list of open problems in the subject.

On the extension problem for uniformly continuous real-valued maps

SEITHUTI MOSHOKOA

(University of South Africa, PRETORIA, South Africa)

(moshosp@unisa.ac.za)

It is well known that uniformly continuous maps between metric spaces admits a unique uniformly continuous extension to the completion. However, there are maps on metric spaces with extensions to the completion which are not uniformly continuous. We give a simple result concerning uniformly continuous maps on bounded subsets which extends the classical Theorem referred to above. Finally a discussion of an analogue of the classical Tietze extension Theorem in this context will also be presented.

Weaker notions of regularity and normality

EVA MURTINOVA

(Charles University, Prague, Czech Republic)

(murtin@karlin.mff.cuni.cz)

2-to-1 closed preimages of ω_1

PETER J. NYIKOS

(University of South Carolina, Columbia, SC 29208)

Although they are a very specialized class of spaces, 2-1 closed preimages of ω_1 have played an important heuristic role in the theory of locally compact and countably compact spaces. Twice already the solution of a problem involving such spaces has made possible major advances in our understanding of countably compact spaces. In 1986, Fremlin's proof

that such spaces always contain copies of ω_1 under the Proper Forcing Axiom (PFA) was the key to a flood of results culminating in Balogh's proofs that the PFA implies that (1) every first countable, countably compact Hausdorff space is either compact or contains a copy of ω_1, and (2) every compact space of countable tightness is sequential. Ten years

later, statement (1) was shown compatible with CH, with 2-1 closed preimages of ω_1 again turning out to capture most of the difficulties involved. Now there is a third set of problems

to which it is hoped that these special spaces will provide the needed insight: Is it consistent that every locally compact, hereditarily strongly collectionwise Hausdorff space is normal? collectionwise normal? countably paracompact? The answers are negative under $\diamondsuit$ but the PFA is a reasonable candidate for positive answers.

Weakly Whyburn spaces of continuous functions on ordinals

Jan Pelant

(Mathematical Institute of Acad. Sci. of Czech Rep., Žitná 25,115 67 Prague 1, Czech Republic)

(pelant@math.cas.cz)

joint research with Angelo Bella

We characterize those ordinals ξ for which $C_p(\xi)$ is a weakly Whyburn space. As a byproduct we obtain the coincidence of the Fréchet property and the Whyburn one on $C_p(\xi)$.

On the spectrum of χ of ultra-filters on ω

Saharon Shelah

(Hebrew University of Jerusalem)

(shelah@math.huji.ac.il)

We investigate what this set of cardinals can be (when the continuum is large, naturally)as well as $\pi\chi$ continuing Brendle Shelah [BnSh642]. In particular, those sets may be non-convex (see problem 5 there).

Cardinal sequences of scattered spaces

Lajos Soukup

(Alfréd Rényi Institute of Mathematics)

(soukup@math-inst.hu)

joint research with I. Juhász and Z. Szentmiklóssy

We show that if we add any number of Cohen reals to the ground model then, in the generic extension, a locally compact scattered space has at most $(2^{\aleph_0})^V$ many levels of size ω.

We also give a complete ZFC characterization of the cardinal sequences of regular scattered spaces. Although the classes of the regular and of the 0-dimensional scattered spaces are different, we prove that they have the same cardinal sequences.

Erdős space(s)

JAN VAN MILL

(Vrije Universiteit, Amsterdam)

(vanmill@cs.vu.nl)

Let E denote Erdős space, i.e., the set of all vectors in ℓ^2 all coordinates of which are rational. The irrational version of E, i.e., the set of all vectors in ℓ^2 all coordinates of which are irrational, is denoted by E_c. It is known that both E and E_c are 1-dimensional and totally disconnected.

Theorem (Dijkstra, van Mill, Steprāns). *E_c is not homeomorphic to its countable infinite product.*

Theorem (Dijkstra, van Mill). *E is homeomorphic to its countable infinite product.*

If X is a space then $\mathcal{H}(X)$ is the group of all homeomorphisms of X with the compact-open topology.

Theorem (Dijkstra, van Mill). *Let $X = \mathbb{R}^n$, $n \geq 2$, and let D be a countable dense subset of X. The subgroup $\{h \in \mathcal{H}(X) : h(D) = D\}$ of $\mathcal{H}(X)$ is homeomorphic to E.*

Partitioning Topological Spaces - 60 Theorems

W. WEISS

(University of Toronto)

(weiss@math.toronto.edu)

We look back at some of the theorems about partitioning topological spaces and consider some of the remaining open problems. After a brief look at the higher dimensional cases we concentrate on the one dimensional case. We show how to use Cohen reals to partition metric spaces.

Large cardinals below first measurable cardinal

SZYMON ZEBERSKI

(Institute of Mathematics, Wrocław University, , Poland)

(szebe@math.uni.wroc.pl)

joint research with JACEK CICHOŃ

We use the following notation. $M(\kappa)$ denotes κ is a measurable cardinal; $SI(\kappa) - \kappa$ is strongly inaccesible; $WC(\kappa) - \kappa$ is weakly compact.

We are interested in three classes of cardinals: indescribable cardinals, subtle cardinals and partition cardinals. Let us recall the folowing definition.

Definition . Cardinal number κ is Π^1_n–indescribable ($\Pi^1_n - IND(\kappa)$) iff for every ϕ which is Π^1_n–sentence (of a language with one predicate A), for every subset A of R_κ if $(R_\kappa, A, \in) \models \phi$ then there exists $\alpha < \kappa$ such that $(R_\alpha, A \cap R_\alpha, \in) \models \phi$.

It is well known that $\Pi^1_0 - IND(\kappa) \iff SI(\kappa)$ and $(\Pi^1_1 - IND(\kappa) \iff WC(\kappa)$.
For Π^1_n–indescribable κ let
$\mathcal{D}_{\Pi^1_n}(\kappa) = \{X \subseteq \kappa : (\forall \phi \in \Pi^1_n)(\forall A \subseteq R_\kappa)((R_\kappa, A, \in) \models \phi \longrightarrow$
$(\exists \alpha \in X)(R_\alpha, A \cap R_\alpha, \in) \models \phi)\}$.

The second class of caldinals we are interested in are subtle cardinals.

Definition . Let $\mathcal{A} \subseteq P(\kappa)$. We say that κ is $\mathcal{A}$–subtle iff for every sequence $(A_\alpha)_{\alpha\in\kappa}$ such that $(\forall \alpha \in \kappa)(A_\alpha \subseteq \alpha)$, for every closed unbounded set $C \subseteq \kappa$ there exists a set $X \in \mathcal{A}$ such that $X \subseteq C$ and

$$(\forall \alpha, \beta \in X)(\alpha < \beta \rightarrow A_\alpha = A_\beta \cap \alpha).$$

Definition . Let κ be a cardinal number.

1. κ is subtle ($ST(\kappa)$) iff κ is $[\kappa]^2$–subtle.

2. κ is almost ineffable ($AINF(\kappa)$) iff κ is $[\kappa]^\kappa$–subtle.

3. κ is Π^1_n–subtle ($\Pi^1_n - SB(\kappa)$) iff $\Pi^1_n - IND(\kappa)$ and κ is $\mathcal{D}_{\Pi^1_n}$–subtle.

The third class are partition cardinals.

For an ordinal number α let

$$\eta_\alpha = \min\{\lambda : \lambda \rightarrow (\alpha)^{<\omega}\}.$$

Definition . Let κ be a cardinal number.

1. κ is an Erdös cardinal ($E(\kappa)$) iff there exists a limit ordinal α such that $\kappa = \eta_\alpha$.

2. κ is a Ramsey cardinal ($R(\kappa)$) iff $\kappa \rightarrow (\kappa)^{<\omega}$.

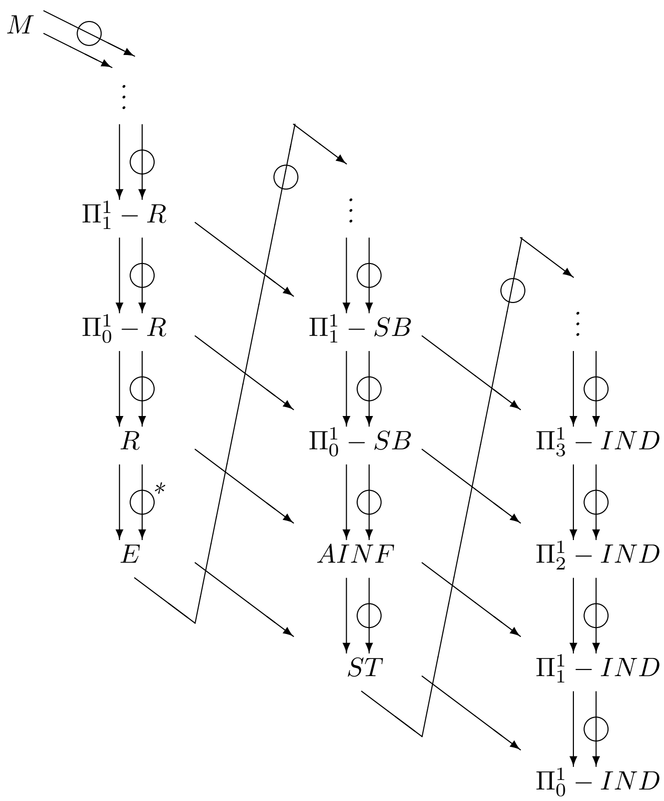
3. κ is a Π^1_n–Ramsey cardinal ($\Pi^1_n - R(\kappa)$) iff $\Pi^1_n - IND(\kappa)$ and

$$(\forall F : [\kappa]^{<\omega} \mapsto 2)(\exists H \in \mathcal{D}_{\Pi^1_n}(\kappa))(\forall n \in \omega)(|F([H]^n)| = 1).$$

We write $A \longrightarrow B$ if $A(\kappa)$ implies $B(\kappa)$.
$A \mathrel{-\!\!\ominus\!\!\rightarrow} B$ denotes that if $A(\kappa)$ then $\{\lambda < \kappa : B(\lambda)\}$ is stationary in κ.
$A \mathrel{-\!\!\ominus\!\!\overset{*}{\rightarrow}} B$ means that if $A(\kappa)$ then $\{\lambda < \kappa : B(\lambda)\}$ has cardinality κ.

Our results sum up the following theorem.

Theorem . *We have the following diagram*



References

[1] W.Boos, *Lectures on large cardinal axioms,* Proccedings of the International Summer Institute and Logic Colloquium, Kiel 1974.

[2] F.Drake, *Set theory, an introduction to large cardinals,* North Holland 1974.

[3] A.Kanamori, The Higher Infinite, Springer-Verlag Berlin Heidelberg 1994.

[4] K.Kunen, *Indescribability and the continuum,* Proccedings of Symposia in Pure Mathematics, Volume XIII, Part I, pp.199-203.

Key words and phrases. Measurable cardinal; Π^1_n–indescribable cardinal; stationary set.

2000 AMS Subject Classification. 03E55, 03E10.