$$P\left\{\bar{\mathbf{x}} \in \left(\mu - \frac{2h|\bar{\mathbf{x}}-\mu|}{N} \middle| \leqq x \leqq \mu + \frac{2h\,|\,\bar{\mathbf{x}}-\mu\,|}{N}\right) \middle| \mu, \sigma\right\} \geqq .95+\beta-1=\beta-.05$$

Again, $I_{\mathscr{J}}\ [\theta, \hat{\varphi}\ (\boldsymbol{E}, \theta)]$ is the point set defined by the inequalities in the "( )". $J_{\Omega_1}\ [\boldsymbol{E}] = J_\mu\ [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n]$ is found by taking for any particular value of $\bar{\boldsymbol{x}}$, the values of μ satisfying these inequalities. The confidence coefficient associated with $J_\mu[\boldsymbol{E}]$ is $\geqq \beta - .05$.

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ON A NEW AXIOMATIC FOUNDATION OF THE THEORY OF PROBABILITY

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The author has given a short account of a new axiomatic foundation of the theory of probability, in which the basic concept is that of conditional probability, and absolute probabilities may not exist. The new theory is a generalization of that of A. N. Kolmogorov*, and includes such schemes which could not be fitted in any of the existing theories, and which have important applications especially in physics.

Let H denote a set, T_1 a Borel field of subsets $A, B, C, \ldots$ of H, called events. Let further a non-void subset T_2 of T_1 be given, and a set function of two variables $P\ (A|B)$ defined for $A \in T_1$ and $B \in T_2$, which satisfies the following axioms:

I. $0 \leqq P\ (A|B) \leqq 1$ and $P\ (B|B) = 1$.

II. For any fixed $B \in T_2$ $P\ (A|B)$ is a σ-additive set function on T_1.

III.** $P\ (A|BC).\ P\ (B|C) = P\ (AB|C)$.

We call $P\ (A|B)$ the conditional probability of A relative to B and $F = (H, T_1, T_2, P\ (A|B))$ a conditional probability field (c.p.f.). A c.p.f. is thus a set of probability fields (p.f.) in the sense of the theory of Kolmogorov which are connected by axiom III. If H is a set, T_1 a Borel field of subsets of H,

*) The author has been informed that in a lecture held some years ago Kolmogorov himself mentioned the possibility of such a generalization of his theory but he did not publish his ideas on this subject.

**) AB denotes the intersection of the sets A and B.

$\mu(A)$ a σ-additive measure on T_1, further if T_2 denotes the set of those $B \in T_2$ for which $0 < \mu(B) < +\infty$, then putting
$P(A|B) = \frac{\mu(AB)}{\mu(B)}$, clearly, $F = (H, T_1, T_2, P(A|B))$ is a c.p.f. If $H \in T_1$ and $\mu(H) = 1$, F is called the c.p.f. generated by the p.f. $(H, T_1, \mu(A))$. If $\mu(A)$ is not bounded, F can not be generated by a p.f.

Generalizations of the laws of large numbers as well of the central limit theorem for a c.p.f. have been found by the author. We mention here only the following consequence of the strong law of large numbers for a c.p.f., which is a generalization of a well known theorem of Borel on normal decimals:

Let us consider the development of a real number x $(0 < x < 1)$ into a Cantor series

$$x = \sum_{n=1}^{\infty} \frac{e_n(x)}{q_1 q_2 \cdots q_n}$$

where the „digits" $e_n(x)$ can take one of the values $0, 1, 2, \ldots q_n - 1$, and $q_n \geqq 2$ is a sequence of integers for which $\lim_{n\to\infty} q_n = +\infty$ and $\sum_{n=1}^{\infty} \frac{1}{q_n} = +\infty$. Let us denote by $f_n(x; k_1, k_2, \ldots, k_s)$ the number of those digits $e_r(x)$ $(r = 1, 2, \ldots, n)$ which are equal to one of the non-negative integers $k_1, k_2, \ldots, k_s$. Then we have for any $s \geqq 2$ and any choice of the non-negative integers $k_1, k_2 \ldots, k_s$ and for almost every x in (0.1) the relation

$$\lim_{n\to\infty} \frac{f_n(x; k_j)}{f_n(x; k_1, k_2, \ldots, k_s)} = \frac{1}{s} \quad (j = 1, 2, \ldots, s).$$

Thus the conditional relative frequencies of the digits are for almost every x in the limit equal for any finite collection of digits.

It can be shown further that for some types of Markov chains and other stochastic processes in cases when there does not exist a limiting distribution in the ordinary sense, there exists a conditional limiting distribution in the sense of the theory sketched above.

All these results will be published in a paper in print in the Acta Mathematica of the Hungarian Academy of Sciences.

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