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If $G = (V, E)$ is a graph then $V = V(G)$ and $E = E(G)$ are the vertex set and the edge set of G, resp. An edge $\{x, y\}$ is said to join the vertices x and y and denoted by xy.

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If $xy \in E(G)$ then x and y are said to be *adjacent* (in G) and the vertices x and y are *incident* with the edge xy.

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The set of vertices adjacent to a vertex $V \in V(G)$ *is denoted by* $\Gamma_G(v)$ *or* $N_G(v)$*. The degree of a vertex* v *is given by* $D_G(v) = |\Gamma_G(v)|$*.*

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Since each edge has two endvertices (the vertices incident to the edge), the sum of the degrees is exactly twice the number of edges:

$$\sum_{v \in V(G)} d_G(v) = 2|E(G)| \ .$$

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- *they have the same number of connected components*

Connectedness

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A walk in a directed or undirected graph is a sequence $(x_0, e_1, x_1, e_2, x_2, \ldots, x_{k-1}, e_k, x_k)$ *in which* $x_0, x_1, \ldots, x_k$ *are vertices and* e_i *is an edge from* x_{i-1} *to* x_i $(i = 1, 2, \ldots, k)$*. The length of the walk above is* k*.*

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A path is a trail and a trail is a walk.

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Proposition

If two vertices of a graph are connected by a walk, they are also connected by a path

Connectedness cont'd.

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Connectedness of the vertices is an equivalence relation.

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The equivalence classes are called the (connected) components of the graph. A graph is connected if it consists of one connected component.

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Definition

A walk $(x_0, x_1, x_2, \ldots, x_{k-1})$ is a circuit or cycle if $x_0, x_1, x_2, \ldots, x_{k-1}$ are distinct vertices and $x_k = x_0$.

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The complement of a graph G is the graph $\overline{G}$ defined by $V(\overline{G}) = V(G)$, $E(\overline{G}) = \{xy \colon x, y \in V(G), x \neq y, \;\; xy \notin E(G)\}$.

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Definition

The complete graph of n vertices, denoted K_n, is the graph in which every pair of vertices is joined by an edge.

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Notice that a graph G is bipartite if and only if $\chi(G) \leq 2$.