PIGEONHOLE PRINCIPLE

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PIGEONHOLE PRINCIPLE, cont'd

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PIGEONHOLE PRINCIPLE, example

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Question

There are 15 minicomputers and 10 printers in a computer lab. At most 10 computers are in use at one time. Every 5 minutes, some subset of computers requests printers. We want to connect each computer to some of the printers so that we should use as few connections as possible but we should be always sure that a computer will have a printer to use. (At most one computer can use a printer at a time.) How many connections are needed?

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Note that if there are fewer than 60 connections then there will be some printers connected to at most 5 computers. If the remaining 10 computers were used at one time, there would be only 9 printers left for them. Thus, at least 60 connections are required.

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On the other hand, it can be shown that if the i-th printer is connected to the i-th, $(i+1)$-st, …, $(i+5)$-th computers $(i = 1, \ldots, 10)$ then these 60 connections have the desired properties.

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$p_1 \quad p_2 \quad p_3 \quad p_4 \quad p_5 \quad p_6 \quad p_7 \quad p_8 \quad p_9 \quad p_{10}$

$c_1 \quad c_2 \quad c_3 \quad c_4 \quad c_5 \quad c_6 \quad c_7 \quad c_8 \quad c_9 \quad c_{10} \quad c_{11} \quad c_{12} \quad c_{13} \quad c_{14} \quad c_{15}$

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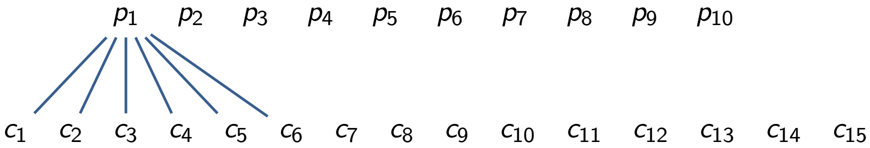

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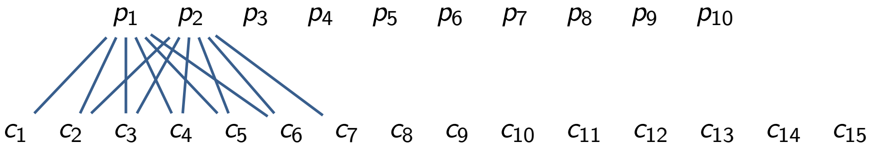

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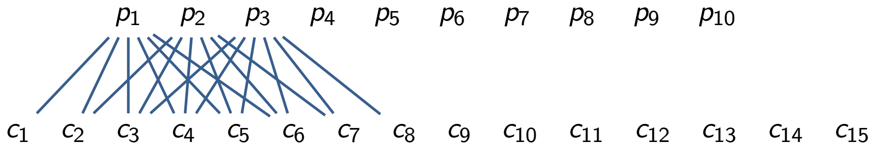

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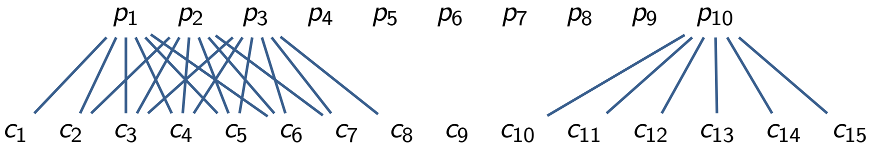

PIGEONHOLE PRINCIPLE, 2nd example

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Show that if $n + 1$ *numbers are selected from the set* $\{1, 2, 3, \ldots, 2n\}$ *then one of these will divide another one of them.*

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Take n "pigeonholes". Put the selected numbers

of form $(2k-1)2^{\alpha}$ into the k-th pigeonhole $(1 \leq k \leq n)$. Then at least one pigeonhole will contain at least two numbers and one of these will divide another one of these.