$$a_n = a_{n-1} + \frac{n(n-1)}{2} + 1 \qquad a_0 = 1$$

$$g(x) = \sum a_n x^n$$

$$\sum_{1} a_n x^n = \sum_{1} a_{n-1} x^n + \sum_{2} \left(\frac{n(n-1)}{2} + 1 \right) x^n$$

$$g(x) - 1 = x\,g(x) + \underbrace{\sum_{2} \binom{n}{2} x^n}_{x^2 \underbrace{\sum_{2} \binom{n}{2} x^{n-2}}_{\frac{x^2}{(1-x)^3}}} + \underbrace{\sum_{1} x^n}_{\frac{1}{1-x} \cancel{-1}}$$

$$(1-x)\,g(x) = \frac{x^2}{(1-x)^3} + \frac{1}{1-x}$$

$$g(x) = \frac{x^2}{(1-x)^4} + \frac{1}{(1-x)^2} = x^2 \sum_{0} \binom{n+3}{3} x^n + \sum_{0} \binom{n+1}{1} x^n =$$

$$= \sum_{0} \binom{n+3}{3} x^{n+2} + \sum (n+1) x^n = \sum_{2} \binom{n+1}{3} x^n + \sum (n+1) x^n$$

$$a_n = \frac{(n+1)(n)(n-1)}{6} - (n+1) = \frac{1}{6} n^3 + \frac{5}{6} n^2 + 1$$