HOMEWORK SET #2 / CO1A / Spring 2020

1. Prove that $\sum_{k=1}^{n} \frac{(-1)^{k-1}}{k}\binom{n}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$.
2. Show (preferably by a combinatorial argument) that $S(n,k) = kS(n-1,k) + S(n-1,k-1)$.
3. Show by a combinatorial argument that
$$S(n+1,k) = \binom{n}{0}S(0,k-1) + \binom{n}{1}S(1,k-1) + \cdots + \binom{n}{n}S(n,k-1).$$
4. Compute $S(n,4)$, S$(n, n-3)$ and S$(n, n-4)$.
5. In how many different ways can you reach the bottom-right corner of a chessboard from the top-left corner, if you may only step right and downward and you may never be above the top-left bottom-right diagonal of the board (hint: think of the Pascal triangle)
6. Find the ordinary generating function of the sequence n^2, i.e. find a close formula for the function $\sum_{n=0}^{\infty} n^2 \cdot x^n$.
7. Find the sequence (a_k) such that the function $\frac{1}{1+x^4}$ is the ordinary generating function of (a_k), that is $\frac{1}{1+x^4} = \sum_{n=0}^{\infty} a_n \cdot x^n$.
8. Find the sequence (b_k) such that the function $\frac{b}{a+x}$ is the ordinary generating function of (b_k), that is $\frac{b}{a+x} = \sum_{n=0}^{\infty} b_n \cdot x^n$.

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