HOMEWORK SET #4 / CO1A / Spring 2020

1.) Solve the recurrence relation under the given initial conditions: $a_k = 6a_{k-1} - 11a_{k-2} + 6a_{k-3}$, $a_0 = 1,\ a_1 = 4,\ a_2 = 9$.

2.) Remember that the Fibonacci numbers are defined by $F_n = F_{n-1} + F_{n-2}$ and $F_0 = F_1 = 1$. Find closed formulas for $F_0 + F_2 + F_4 + \cdots + F_{2n}$, $F_1 + F_3 + F_5 + \cdots + F_{2n+1}$ and $F_0 + F_1 + F_2 + \cdots + F_n$ $(n \geq 1)$ (you may express them sums in terms of some — fixed number of — members of the Fibonacci sequence).

3.) Determine a recurrence for $f(n)$, the number of regions into which the space is divided by n planes in general position (and for that define the general positions of the planes as well such a way that they will give the highest possible numbers of parts of the plane). Solve the recurrence to obtain the number of parts.

4.) How many subsets does the set $\{1, 2, \ldots, n\}$ have that contain no two consecutive integers?

5.) Solve the recurrence relation under the given initial conditions: $c_n = 9c_{n-1} - 15c_{n-2} + 7c_{n-3}$, $c_0 = 0,\ c_1 = 1,\ c_2 = 2$.

6.) We have n forints. Every day we buy exactly one of the following products: pretzel (1 forint), candy (2 forints), icecream (2 forints). What is the number of possible ways of spending all the money (the order of the bought products counts)?

7.) Remember that the Fibonacci numbers are defined by $F_n = F_{n-1} + F_{n-2}$ and $F_0 = F_1 = 1$. Prove that for every pair $m \geq 0,\ n > 0$ $F_{n+m+1} = F_{n-1}F_m + F_nF_{m+1}$.

8.) Prove that the number of partitions of a number n into no more than r terms is equal to the number of partitions of n into any number of terms, each at most r. (A partition of a number n into r terms is a collection of positive integers $\{a_1, a_2, a_3, \ldots, a_r\}$ such that $a_1 + a_2 + \cdots + a_r = n$; note that order does not count!)

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