HOMEWORK SET #5 / CO1A / Spring 2020

1.) Solve the recurrence relation under the given initial conditions: $k_n = k_{n-1} + n + 6$, $k_0 = 0$.

2.) Determine a recurrence for $f(n)$, the number of regions into which the plane is divided by n circles in general position (circles are in general position if each two intersect in two points and no three share a common point). Solve the recurrence to obtain the number of parts.

3.) Solve the recurrence relation under the given initial conditions: $a_{n+2} = 2a_n - a_{n+1} + 3 \cdot (-2)^n$, $a_0 = -1$, $a_1 = 1$.

4.) Solve the recurrence relation under the given initial conditions: $h_{k+2} = 2h_{k+1} - h_k + 2^k$, $h_0 = 2$, $h_1 = 1$.

5.) Find both the recurrence relation and the appropriate generating function for the number of n-digit numbers (over $\{0, 1, \ldots, 9\}$) where digit 0 can be used only even number of times. Use the generating function to find the number!

6.) Let D_n denote the number of derangaments of n objects. Let C_n be defined by

$$C_n = \frac{D_n}{n!} - \frac{D_{n-1}}{(n-1)!}.$$

Find a recurrence for C_{n+1} in terms of C_n, solve it (using iteration) and use expression for C_n to express D_n.

7.) Prove that $x^n = \sum_{k=1}^{n} S(n,k)x(x-1)\cdots(x-k+1)$ (Hint: prove that the two polynomials on the two sides of the equation are equal at values $x = 0, 1, 2, \ldots, n$.)

8.) For each of the following functions, find the sequences for which the function is the ordinary and (for a different sequence) the exponential generating function: $\frac{1}{1-x} + e^{6x}$, $(1+x^2)^n + 1$, $e^{x/2} + x^2 + x^3$

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