

HOMEWORK SET #7 / Sample midterm / CO1A/ Spring 2020

as was the MIDTERM / CO1A / Fall 2019

- 1.) In how many ways can we choose 8 bottles of soda if there are 12 brands available, if
 - a.) order counts and repetition is allowed?
 - b.) order does not count and repetition is allowed?
 - c.) order counts and repetition is not allowed?
 - d.) order does not count and repetition is not allowed? **(8 points)**
- 2.) How many $n \times m$ matrices do you have (n rows and m columns) whose entries are only 0's or 1's and all rows are different? **(5 points)**
- 3.) How many different ways can exactly one gentleman get his coat back if n of them checked their coat in the cloakroom and they get them back randomly. **(6 points)**
- 4.) Find both the recurrence relation and the appropriate generating function (not from the RR) for the number of n -digit numbers (over $\{0, 1, \dots, 9\}$) where digit 0 can be used only even number of times. Use the generating function to find the number! (Note: the n -digit numbers may begin with an arbitrary number of zeroes) **(6 points)**
- 5.) Determine a recurrence for $f(n)$, the number of the regions the 3D space is divided by n balls of general position (i.e. each pair of them intersect in a circle, each three of them have exactly two common points and no four of them have a common point). You do not have to solve the recurrence relation! **(6 points)**
- 6.) Find the ordinary generating function of the sequence n^2 , i.e. find a close formula for the function $\sum_{n=0}^{\infty} n^2 \cdot x^n$. **(5 points)**
- 7.) Suppose that there are $2p$ kinds of objects, each in infinite supply. Let a_k be the number of distinguishable ways of choosing (without order) k objects if only an even number (excluding 0) of each of the first p kinds of object and an odd number of each of the second p kinds of object can be taken. Set up a generating function for the sequence (a_k) and solve for a_k . **(6 points)**
- 8.) You must sit n pairs of policemen around a round table such that no policemen from the same pair are sitting next to each other. How many different ways can you do that? **(6 points)**