HOMEWORK SET #8 / CO1A / Spring 2020

1. Find the number of non-isomorphic connected, unicyclic graphs (graphs with exactly one cycle) on 6 vertices (a bit boring, but useful).
2. Prove that a graph G contains a circuit of length at least $k+1$ if $d(x) \geq k$ for all $x \in V(G)$.
3. Show that the complement of a disconnected graph is connected!
4. Determine all graphs with exactly one pair of vertices of equal degree (all other degrees are distinct).
5. For every $n \geq 3$ give an example of a graph G having $\chi(G) \geq n$ but $G \not\supseteq K_n$.
6. Prove that for every graph G on the vertex set V there is a partition of $V = V_1 \cup V_2$ such that if $G(V_1)$ and $G(V_2)$ denote the graphs spanned by G on the sets V_1 and V_2 respectively, than $\chi(G(V_1)) + \chi(G(V_2)) = \chi(G)$.
7. Prove that a graph has at least $\binom{\chi(G)}{2}$ edges.
8. Prove that for any graph G on n vertices $\chi(G)\chi(\overline{G}) \geq n$ holds.