HOMEWORK SET #9 / CO1A / Spring 2020

1. Give an example of a graph G with $\chi(G) = 2$ and a coloring of G with the greedy algorithm (that is an ordering of the vertices of G in which order the greedy algorithm colors the vertices of the graph) requiring n colors (for every n big enough)
2. Prove that for any graph G on n vertices $\chi(G) + \chi(\overline{G}) \leq n + 1$ holds.
3. What can be the chromatic number of a connected simple graph with exactly two cycles?
4. Show that a graph G is a tree if and only if

   it is cycle-free, but adding any edge to G will create a cycle.
5. Prove that in every tree on n vertices there are at least $\frac{2n+2}{3}$ vertices of degree less than four. For every $n = 3k + 2$ give a tree where the number of vertices of degree less than four is exactly $\frac{2n+2}{3} = 2k + 2$.
6. Show that in a connected graph every two maximum (length) paths have a common vertex.
7. Find the labeled tree on 9 vertices (with labels: 1 through 9) with Prüfer code 6,2,1,6,2,9,8.
8. Show that for every sequence $1 \leq d_1 \leq d_2 \leq \cdots \leq d_n$ with $n \leq \sum_{i=1}^{n} d_i \leq 2n - 2$, $2 \big| \sum_{i=1}^{n} d_i$ there is a forest with this sequence as degree sequence. Is this forest unique (up to isomorphism)?