HOMEWORK SET #10 / CO1A / Spring 2020

1. Prove that any acyclic subgraph of a connected graph can be completed into a spanning tree (i.e. a spanning graph which is a tree) of the graph.
2. Show that a graph G is a tree if and only if

   it is cycle-free, but adding any edge to G will create a cycle.
3. Let G be a graph of n vertices, m edges and k components. Prove that G contains at least $m - n + k$ cycles.
4. A social worker has to make altogether 43 visits, at least one on each day. Is there a period of consecutive days on which he makes exactly 21 visits if he makes his visits on 22 days.
5. Same as problem #4, with 23 days, i.e., assume a social worker has to make altogether 43 visits, at least one on each day. Is there a period of consecutive days on which he makes exactly 21 visits if he makes his visits on 23 days?
6. Show that a sequence of $n^2 + 1$ real numbers always has a (not necessarily strictly) monotone subsequence of length at least $n + 1$. Is the same statement true for a sequence of length n^2?
7. The set M consists of nine positive integers, none of which has a prime divisor larger than six. Prove that M has two elements whose product is a square of an integer.
8. Show that infinitely many members of the sequence

$$1, 11, 111, 1111, 11111, 111111, \ldots$$

   are divisible by $2^{2020} + 1$.