HOMEWORK SET #12 / CO1A / Spring 2020

as was the FINAL / CO1A / Spring 2019

The maximum number of points given to any problem is indicated at the end of the problems in **(boldface)**. *The maximum number of points given to the whole final would be 72, but now is only 65, as problem 9 is listed here just for your information, but NOT TO BE SOLVED as the background material is has not been covered yet*

1.) Prove that

$$\frac{(m+n)!}{m!n!} \leq \frac{(m+n)^{m+n}}{m^m n^n}$$

Hint: use binomial theorem! **(7 points)**

2.) Ramsey quiz: Find the following Ramsey numbers (without justification, just give the values): $R(P_4, C_6)$, $R(C_4, C_5)$, $R(K_3, C_6)$. **(3+5+4=12 points, but -1 point for each bad answer!!!!)**

3.) Is there any graph isomorphic to its complement on n vertices for $n = 4, 5, 6$? **(6 points)**

4.) Suppose that $a_{n+1} = (n+1)b_n$, with $a_0 = b_0 = 1$. Let $A(x)$ denote the exponential generating function of the sequence (a_n) and $B(x)$ denote the exponential generating function of the sequence (b_n). Derive a relation between $A(x)$ and $B(x)$. **(6 points)**

5.) In how many ways can one sit n couples around a round table alternating if no spouses from the same couple can be seated next to each other. **(8 points)**

6.) In the Hungarian TOTO (Italian Calcio) game one may bet on the outcome of the soccer games: 1 for the win of the first team, 2 for the win of the second team, x for a draw. How many tickets do you have to fill if there are three matches to bet on and you surely want to have a ticket which misses all the final results? **(10 points)**

7.) A social worker has to make altogether 60 visits, at least one on each day. Is there a period of consecutive days on which he makes exactly 15 visits if he makes his visits on a) 31 days b) 30 days? **(9 points)**

8.) You have n forints. Every day you buy exactly one of the following products: pretzel (1 forint), candy (2 forints), ice-cream (2 forints), soda (2 forints), bubble gum (2 forints), cracker (2 forints), chocolate bar (3 forints), mineral water (3 forints) or sandwich (3 forints). What is the number of possible ways of spending all the money (the order of the bought products counts)? **(7 points)**

9.) **Listed only for the sake integrity, but not to be solved!!** Prove that in a (b, v, k, r, λ) balanced incomplete block design over a set of size v, each block having size k and every pair of elements is in exactly λ blocks all points are in the same number of blocks (denoted by r) and derive a formula for r in terms of v, k, λ. **(7 points)**