HOMEWORK SET #8 / CO1A / Spring 2014

1. Find the labeled tree on 9 vertices (with labels: 1 through 9) with Prüfer code 6,2,2,6,2,9,9,9
2. Show that a graph G is a tree if and only if
   it is cycle-free, but adding any edge to G will create a cycle.
3. Prove that in every tree on n vertices there are at least $\frac{2n+2}{3}$ vertices of degree less than four. For every $n = 3k+2$ give a tree where the number of vertices of degree less than four is exactly $\frac{2n+2}{3} = 2k+2$.
4. Show that in a connected graph every two maximum (length) paths have a common vertex.
5. Prove that if $d(x) \geq 3$ for all $x \in V(G)$ then G contains a cycle of even length.
6. Prove that a graph has at least $\binom{\chi(G)}{2}$ edges.
7. Prove that for any graph G on n vertices $\chi(G)\chi(\overline{G}) \geq n$ holds.
8. Prove that for any graph G on n vertices $\chi(G) + \chi(\overline{G}) \leq n+1$ holds.

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