

## HOMEWORK 1

1\*\*. Find a connected graph and 9 longest paths in it such that these paths do not share a common vertex. (Due later!)

2. Show that the longest paths in a tree share a common vertex.

3. Show that the maximum size of a set of integers in  $\{1, 2, \dots, n\}$  not containing two elements whose product is a full square is exactly the number of square-free numbers up to  $n$ .

4. Show that, in a finite affine plane of  $p^2$  points ( $p$  is a prime)

(a) any two points are contained in exactly one line

(b) any two lines has at most one common point.