Page 1

First I formulate the problem I am speaking about.

We have a sequence of i.i.d. random variables $\xi_1, \xi_2, \dots, \xi_n$ on a space X and a class of functions with some good properties defined on the same space X. We define with their help the random variables $f(\xi_i)$ and their normalized sums $S_n(f)$ in the form as it is written down on this page.

We want to give a good estimate on the tail distribution of the supremum of the normalized random sums $S_n(f)$ if we take the supremum for all functions f in our class of functions.

Page 2

The normalized random sums $S_n(f)$ we are considering are asymptotically normal. Hence it is natural to consider a Gaussian version of our problem, where we estimate the supremum of a class of Gaussian random variables. This Gaussian problem can be solved.

The result and method of proof applied in the solution of this Gaussian problem are also valid for non-Gaussian random variables if they satisfy formula (1) on this page.

The question arises whether this observation helps us in solving our original problem.

Page 3

In our problem formula (1) holds only under some slight restrictions. Our main problem is to overcome the difficulties caused by these restrictions. To settle this problem we have to impose new conditions and to find new methods.

These new conditions are formulated with the help of a notion called the class of functions with polynomially increasing covering numbers. I give its definition in two steps on the next page.

Page 4

Here is this definition. I do not read it out, I only explain its main content. We demand in it that the class of functions we are working with has an arbitrarily dense subset with relatively few elements in the $L_1(\nu)$ norm for all probability measures ν. The main point in this definition is that we demand this property for all probability measures.

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In former works our problem was investigated by means of a method called the symmetrization argument. In its application we introduce the randomized version of our random sums. Its definition is explained on this page. Some results enable us to replace the investigation of the supremum of our random sums by the investigation of their randomized versions. But in some cases this method does not work well. It yields a weak estimate if our random variables have a very small second moment. On the other hand, we want to give a good estimate in the general case, which covers such situations, too.

The next theorem, the main result of this paper, provides a useful estimate to achieve this goal.

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Here is this result. Let me remark that in this theorem we work with random sums whose terms may have non-zero expectation. On the other hand, their absolute value is very small.

There are simple examples which show that this result is sharp. In particular, it gives a good lower bound on the levels u for which estimate (2) holds.

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The above 'Main theorem' is interesting for us not as a new inequality, but as a new tool to study our problem. Observe that formula (2) holds in the Main Theorem for relatively large parameters ρ, for parameters that are polynomially decreasing functions of the sample size n. This enables us to work out a new Vapnik–Červonenkis type argument, which yields a better estimate than earlier results.

This better estimate is proved in my paper "Sharp tail distribution estimates for the supremum of sums of i.i.d. random variables".

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Here I could give only a very short explanation of this paper. I wrote a more detailed version which can be found on my homepage at the given address.

I also gave the address of another talk about the solution of the general problem with the help of the Main theorem.

Here I spoke about the problem discussed in this paper and also about earlier approaches to solve it. I tried to explain the novelty value of my result, and the reason why it may give a better estimate than previous methods.

Thank you for your attention.