Page 1

First I describe the problem of this talk.

We define a set of random sums $S_n(f)$ with the help of a sequence of i.i.d. random variables and a class of functions with some good properties in the way as it is written down on this page. These random sums are indexed by the elements of our class of functions.

For all positive numbers v we want to give a good bound on the probability that the supremum of our random sums $S_n(f)$ is larger than this number. We take the supremum for all functions f in our class of functions.

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By some classical results there is a concentration point v_0 with the following property. The above probability is almost one if v is smaller than v_0, while it begins to decrease fast for v greater than v_0. We want to find a good estimate on this concentration point.

Our problem has a natural Gaussian version about the estimation of the supremum of Gaussian random variables. This Gaussian problem is solved. Under some restrictions a similar estimate holds in our problem too. Our goal is to find a general solution which also holds without these restrictions.

To formulate these results I recall the notion of classes of functions with polynomially increasing covering numbers.

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Here is this definition. I do not read it out. I only remark that this notion is a natural version of Vapnik–Červonenkis classes if we are working with a class of functions and not with a class of sets.

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First I show a simple example that may help to understand the behaviour of the concentration point v_0.

In this example we take i.i.d. random variables with uniform distribution on the unit interval $[0,1]$. To define our class of functions we fix a number σ, and first we consider the indicator functions of disjoint subintervals of the interval $[0,1]$ with length σ. We take as many of them as it is possible. Then we define our class of functions by taking the normalized versions of these indicator functions. In such a way we get a model that satisfies our conditions. We want to give a good lower bound on the concentration point v_0 in this model.

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The result on this page gives a good estimate on v_0. The number hat $u(\sigma)$ is a good lower bound for it. I explain the content of this result on the next page.

Page 6

In this result we considered three different cases. In case three we have a large parameter σ, and we get such an estimate for hat $u(\sigma)$ as in the Gaussian case. In case two σ is smaller. In this case a Poissonian and not a Gaussian approximation gives the right choice for hat $u(\sigma)$. Finally, in case one σ is very small, and a trivial consideration gives the right bound on hat $u(\sigma)$.

Next I formulate the main result of this paper in a Theorem and in its Extension. Their right formulation could be found by means of the above example.

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The theorem is formulated here. Three different cases appear in it, similarly to the result about our Example. The result in cases one and two is satisfactory, but to get a complete picture in case three we need an additional result. This is formulated in the Extension of the Theorem on the next page.

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I discuss the content of these results. In cases one and two the theorem itself gives a good estimate. It holds for numbers v suggested by our example. It is a non-Gaussian bound suggested by Bennett's inequality. In case three the theorem and its extension together give a good estimate in the right domain. In the extension we get a good Gaussian estimate if the number v is not too large, while in the remaining cases the original theorem gives the right bound.

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Here I gave a short overview of this paper. A more detailed version can be found on my homepage at the given address. It also explains the main ideas of the proof.

This longer version explains the ideas behind the proof. The most important of them is the application of a new Vapnik–Červonenkis type argument to control the irregular contributions to the supremum we investigate. Actually, the main subject of this reseach was to find this new method.

Thank you for your attention.