Personal Reminiscences, Gyuri Elekes

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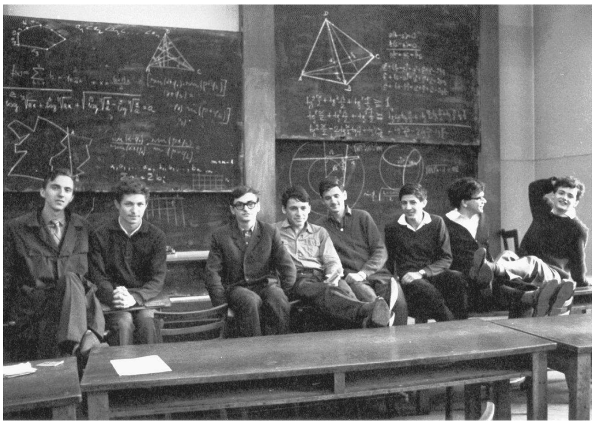
Our friend, Gyuri Elekes[1] was already very ill when we, (Gyuri, Endre Szabó and I) have finished our paper [2]. He died within a month. *This* paper is written in his memory, and has a continuation [5] describing our joint work, partly published in [2] and also a basic ingredient of the proofs in [2] from a paper of Elekes and Szabó [3].



I have known Gyuri Elekes for years and have many kind memories of him. Here I start with a photo, back from his high school years. This shows Elekes during some preparation for the International Mathematical Olympiad.

This photo shows him and his mates: Z. Laborczi, A. Szűcs, L. Csirmaz, Gy. Elekes, L. Babai, J. Pintz, L. Surányi, and Gy. Hoffman. The left photo is the enlarged middle part of the right one. Elekes must have liked this photo, since a printed version of this could be seen on the wall in his office, at Eötvös University.

[1]More precisely, György Elekes

1. Old times

At the University he was one of the most talented students among his peers. When he finished his university studies, in 1972, Vera T. Sós "invited him" to become a lecturer at the Department of Analysis I, Eötvös University, Budapest. This is when we became colleagues,[2] friends, though I knew him a already from his student years. Ákos Császár was the head of the department, Vera T. Sós, Kató Rényi, András Hajnal, Rózsa Péter, Lajos Pósa, Miklós Laczkovich were also working here.

The department was otherwise fairly small, fluctuating, but basically ten permanent persons.[3] We taught primarily Mathematical Analysis, Theory of Function, Combinatorics, Graph Theory, Set Theory, Mathematical Logic, Approximation Theory, Topology, etc. (There was also another department in Analysis, teaching the other Analysis-related subjects, e.g., Differential Equations, PDE, Functional Analysis, Fourier Series, etc.)

There were a lot of tensions, frictions between the distinct departments. However, our Department was extremely friendly. We enjoyed this friendly atmosphere very much, and also were proud to work at mathematically such an excellent department. And, beside being enthusiastic mathematicians, we also enjoyed very much the teaching, among others, the contact with outstanding young students. We worked together until 1985, here in the same group (of approximately 10 people). Then, for some strange reasons[4] I left the University, while he stayed.

Those days I had to think over, what had I felt sorry for to leave behind. I felt sorry to leave the teaching of excellent students, some of my dreams, to leave behind some of my best colleagues and friends, including Elekes. Let me describe here, how do I remember him.

- He was very talented, fast, clear-thinking;
- He was also very determined, sometimes tough, however, very kind at the same time;

[2] I was working there since 1967.
[3] The "Mathematics Institute" consisted of 6, later of 7 such departments.
[4] tensions, fights, quarrels

• He was very kind not only because he was very often smiling, but because he was really a kind person.

2. Beginning of the end

When somebody learns that he will soon die, his reactions may characterize him. I do not know, how did Elekes learn that his time was up, but from that on he started finishing his papers.

Within a short period he has finished seven papers. One of them was the paper joint with Endre Szabó [3] and another joint with Endre and me [2].

Beside working very hard, Elekes, of course tried to spend a lot of time with his family and friends as well. They were very important for him.

— ... —

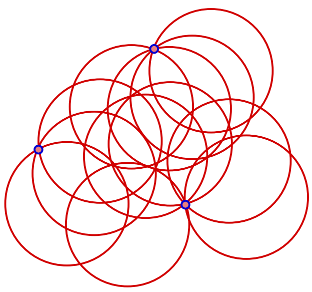
In January 2006 Elekes was doing something in the Rényi Institute, (he often came there) and we started a mathematical discussion. He got stuck in a problem and I suggested a possible approach to overcome this difficulty.

He wanted to prove that some functional relation of the form

$$F(\varphi_1(u,v),\varphi_2(u,v),\varphi_3(u,v)) = 0$$

cannot be satisfied (see [3, 2]) unless F has a very special form. I suggested to examine the singularities of the functions φ_i and derive a contradiction, using that these singularities cannot cancel out each other. (We had similar mathematical discussions several times even earlier, however, this was the first time we seriously started research together.) My idea came partly from a Number Theory book of Paul Turán, partly from my knowledge of function theory.

Within a few days Elekes came back with a ready to publish paper. Do not misunderstand it, the paper was far from being trivial. Its main result was



Theorem 1. *Assume we have three distinct points in the plane, A, B, and C, and we have through each of them n unit circles.*[5] *Then the number of triple points, i.e. points belonging to three such circles, is at most $cn^{2-\eta}$, for some constants $c > 0$ and $\eta > 0$.*

Elekes was interested in this question because if we take straight lines instead of unit circles, then we get a completely different answer:

> Assume we have a square grid arrangement in the plane and we also consider n horizontal, n vertical straight lines, and n of slope 1 (i.e. at 45^o). They will have cn^2 triple points. This can be seen on Figure 1(b). Moreover, using, say, 100 slopes we obtain 100 families of straight lines for which the number of crossings of multiplicity 100 is $c_{100}n^2$.

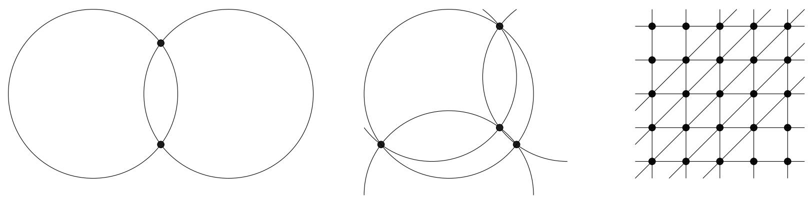


Figure 1: (a) Circles (b) and straight lines

So, this makes a combinatorial distinction between straight lines and unit circles. Of course, if we do not fix the radius of the circles, then such a distinction cannot be made, since any system of straight lines can be transformed into a system of circles, using an inversion.[6]

So the paper was finished, and it was far from being trivial. Yet, I felt the result too special, too narrow. So we have decided to try to find out, what is the more general situation, what is really going on in the background. We wanted to find out, when and why can such systems of curves have cn^2 triple points. We shall see in the second part, that in some sense we, together with Endre Szabó, have succeeded, though this slowed down the publication of the paper by years. But this will be discussed in the next paper [5].

Remark 2. Here I have to point out some slackness in the comment above. I wrote that Elekes wanted to find a combinatorial distinction between straight

[5] From now on, we shall write "$n+n+n$ curves" in such cases.

[6] Or, equivalently, the transformation $w = \frac{1}{z}$, assumed that all the straight lines avoid the origin. The opposite direction does not hold: two points can be contained in arbitrary many circles and this incidence pattern cannot be obtained using straight lines.

lines and unit circles. Several papers of Elekes were connected to this "distinction". Yet, if we try to formulate, what does a combinatorial distinction really mean, then we have to be careful. We could say that whenever we have in $\mathcal{R}^n$ some surfaces and some points, then we can attach to them a bipartite graph $G[\mathbb{S}, \mathbb{P}]$, where $\mathbb{P}$ is the set of points, $\mathbb{S}$ is the family of surfaces, and $S \in \mathbb{S}$ is joined to $P \in \mathbb{P}$ if they are incident. Now we speak about a "combinatorial property" of $\mathbb{S}$ if it can be seen from $G[\mathbb{S}, \mathbb{P}]$.

In this sense we can easily distinguish straight lines and unit circles in the following sense. For straight lines, $G[\mathbb{S}, \mathbb{P}]$ does not contain C_4. For unit circles $G[\mathbb{S}, \mathbb{P}]$ can contain C_4 but cannot contain $K(2, 3)$. So, looking at $G[\mathbb{S}, \mathbb{P}]$ we often can conclude that those curves in $\mathbb{S}$ cannot be straight lines, ... We have several interesting results and deep open problems in this field, however, we will not discussing this topic here.

2.1. Elekes and the Mathematics: "early influences"

Gyuri Elekes and I often discussed mathematics. Occasionally we had different views, and Gyuri was perhaps too modest in the sense that if I asked him, "Why don't you learn this-and this, perhaps that helps in solving your problem", he used to answer this question making the impression that that part of mathematics was to involved for him. The truth was just the opposite: he learnt a lot of new mathematics to proceed with his favorite problems. Then he applied these results in his research and teaching, too.

Let us stop at this point for a minute. I often felt that Elekes was in some positive sense self-certain. When he referred to the difficulty of learning something new, I often felt that he just wanted to go on with his original approach.

Trying to describe Elekes' mathematical carrier, we see that he started with Erdős-Hajnal type combinatorial set theory, then he worked in theoretical computer science, above all in algorithms, and finally he worked in combinatorial geometry, on problems where deep algebraic methods had to be used.

He himself formulated this slightly differently. In one of his CV's he wrote

Field of interest: Combinatorial Geometry and algorithms, combinatorial number theory, and combinatorial algebra. [7]

One of his related lectures on this topic had the title

The interface between geometry, algebra and number theory.

Many of his papers could have this same title.

I close this part with just stating that for many people one of Elekes' result [1] became a crucial one. It is an extremely short and nice gem in mathematics with important consequences, saying that the volume of a high dimensional convex body, given by an oracle, cannot be approximated up to a factor of 2 in less than exponentially many steps. For a longer explanation see the paper of Lovász [4] here.

I finish these reminiscences with that he was a person who loved people and who was a person to be loved.

References

[1] Gy. Elekes, A geometric inequality and the complexity of computing volume. Discrete Comput. Geom. 1 (1986), no. 4, 289–292.

[2] Gy. Elekes, M. Simonovits and E. Szabó, A Combinatorial Distinction between Unit Circles and Straight Lines: How Many Coincidences can they Have? Combinatorics, Probability, Computing, 18 (2009), no. 5, 691–705. [8]

[3] György Elekes and Endre Szabó, How to find groups? (And how to use them in Erdős geometry?), Combinatorica 32 (2012), no. 5, 537571.

[4] L. Lovász on Elekes, here.

[5] M. Simonovits and E. Szabó, Gyuri Elekes and the incidences, (in this volume)

[7] I have to remark here that I have never heard of this last topic before.

[8] This and the next papers were under publication in time my paper was submitted.