

AN EXAMPLE CONCERNING OPEN EVERYWHERE DISCONTINUOUS FUNCTIONS

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In the following, I shall give an example which answers in the affirmative the following problem posed by S. Marcus [2]. Does there exist an open (Lebesgue measurable or even Borelian) everywhere discontinuous function which has the Darboux property in no interval?

Let $F(x)$ be any function which takes on every value in every interval. For the existence of such functions see, for instance, W. Sierpiński [3]. Put

$$f(x) = \begin{cases} F(x), & \text{if } F(x) \neq 0 \\ 1, & \text{if } F(x) = 0. \end{cases}$$

The function $f(x)$ is never 0, but it assumes every value other than 0 in every interval. Therefore $f(x)$ is an open everywhere discontinuous function (we have $f(G) = (-\infty, \infty) - \{0\}$ for every nonvoid open set G), but $f(x)$ has the Darboux property in no interval.

In [1] it was proved, among others, 1° the existence of a Lebesgue measurable function which takes on every value in every interval; 2° the existence of a Borel function which takes on every value in every interval. From 1° it follows, by the above construction, the existence of an open, Lebesgue measurable, everywhere discontinuous function which has the Darboux property in no interval. From 2° it follows the existence of an open, Borelian, everywhere discontinuous function, which has the Darboux property in no interval.

Note added by S. Marcus. The above Note is extracted from a letter of Professor Erdős, sent to me in August 2, 1964. In a letter dated December 7, 1965, Professor Jack G. Ceder (University of California,

Santa Barbara) communicated to me an example of a function from R to R , which is open, of Baire class 2, yet fails to be Darboux on each subinterval and is discontinuous everywhere. This example answers in the affirmative the Problem 4 of my Note [2]. In a letter dated of December 8, 1965, Jack G. Ceder informed me that a similar example is given by Example 6.1 p. 105 of the paper *Darboux continuity* (Jahresbericht der deutschen Mathematiker—Vereinigung, 1965, **67**, 3, 93—117) by A. M. Bruckner and J. G. Ceder. The example of Ceder uses some results of his still unpublished paper in collaboration with M. Weiss: *Some in-between theorems for Darboux functions*, while the example of A. M. Bruckner and J. G. Ceder uses a lemma from a paper of A. M. Bruckner, J. G. Ceder and M. Weiss: *On uniform limits of Darboux functions*, to appear in *Colloquium Mathematicum*.

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2. S. MARCUS, *Open everywhere discontinuous functions*. American Mathematical Monthly 1965, **72**, 9, 993—994.
3. W. SIERPIŃSKI, *Sur une propriété des fonctions réelles quelconques, définies dans les espaces métriques*. Le Matematiche, Catania, 1963, **8**, 73—78.