More detailed results can be obtained in the case where g and h are analytic in u and v.

An Application of Minimal Solutions of Three-term Recurrences to Coulomb Wave Functions*

WALTER GAUTSCHI

Minimal solutions of three-term recurrence relations can be computed accurately by various algorithms based on the theory of continued fractions. These algorithms may be adapted to computing solutions which, though not minimal, do initially behave like a minimal solution and only later, for large arguments, diverge away from the minimal solution. This note exhibits a nontrivial example of a situation of this kind, arising in the recursive computation of Coulomb wave functions.

More specifically, the concern is with computing $\lambda_L = i^L P_L^{(i\eta,-i\eta)}(-i\omega)$, where $P_n^{(\alpha,\beta)}(x)$ denotes the Jacobi polynomial of degree n, and η, ω are real parameters with $\omega > 0$. The sequence $\{\lambda_L\}_{L=0}^{\infty}$ is a solution of the three-term recurrence relation

$$y_{L+1} = \frac{2L+1}{L+1}\omega y_L + \frac{L^2+\eta^2}{L(L+1)} y_{L-1} \quad (L = 1, 2, 3, \ldots),$$

and corresponds to initial values $y_0 = 1$, $y_1 = \omega - \eta$. The recurrence relation, on the other hand, has a minimal solution $\{\lambda'_L\}$ which, though linearly independent of $\{\lambda_L\}$, has the property $\lambda_0 = \lambda'_0$, $\lambda_1 - \lambda'_1 \to 0$ as $\eta \to \infty$. In fact,

$$\lambda_1 - \lambda'_1 = \frac{2\eta}{e^{2\eta\phi} - 1}, \quad \text{where} \quad \phi = \arctan(1/\omega).$$

This information is used to compute λ_L accurately, and leads to an effective algorithm for computing the regular Coulomb wave functions $F_L(\eta, \varrho)$ $(L=0, 1, 2, \ldots)$ over a large region of the parameters η, ϱ.

On the Distribution of Prime Divisors of n**

P. ERDÖS

Denote by $v(n; a, b)$ the number of distinct prime factors of n satisfying $a < p \leqslant b$. $v(n) = v(n; l, n)$ denotes the number of distinct prime factors of n. A well known theorem of HARDY and RAMANUJAN states that for almost all n, $v(n) = (1 + o(1))$ $\log\log n$. The principal aim of this paper is to prove that if $b - a/\log\log\log\log n \to \infty$

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then for almost all integers n

$$v(n; a, b) = (1 + o(1))(\log\log b - \log\log a)$$

uniformly in a and b. More precisely to every $\varepsilon > 0$ there is a c so that if we neglect $o(x)$ integers $n < x$ then for every a, b satisfying

$$\log\log b - \log\log a > c \log\log\log n \tag{1}$$

we have

$$(1-\varepsilon)(\log\log b - \log\log a) < v(n; a, b) < (1+\varepsilon)(\log\log b - \log\log a)$$

We also show that (1) is essentially best possible. The proof uses TURÁN's method and other ideas of probabilistic number theory.

Some related results in probability and number theory are also discussed.

Das Minimum von $D/f_{11}f_{22}\cdots f_{55}$ für reduzierte positive quinäre quadratische Formen*

B. L. VAN DER WAERDEN

For a reduced positive quadratic form $f(x)$ with coefficients f_{ik} a 'fundamental inequality' of the form

$$D \geqq c_n f_{11} f_{22} \cdots f_{nn}$$

was proved by MINKOWSKI. For $n = 1, 2, 3, 4$ the best values of c_n are known:

$$c_1 = 1, \quad c_2 = \tfrac{3}{4}, \quad c_3 = \tfrac{1}{2}, \quad c_4 = \tfrac{1}{4}.$$

The best value of c_5 will be determined in this paper. It is $\frac{1}{8}$. Thus we have, for $n = 5$, the inequality

$$D \geqq \tfrac{1}{8} f_{11} f_{22} \cdots f_{55}.$$

Equality holds for the form $f(x)$ corresponding to the densest lattice packing of five-dimensional spheres.

Some Remarks on a Functional Equation Characterizing the Root**

MAREK KUCZMA

The root functions

$$\varphi(x) = c\sqrt[p]{x} \tag{1}$$

are characterized by the functional equation

$$\varphi(x^{p+1}) = x\varphi(x). \tag{2}$$

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** Received December 14, 1967.