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ON THE FUNDAMENTAL PROBLEM OF MATHEMATICS

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I read with interest the paper of C. Goffman, "And What is your Erdös Number?" (this MONTHLY, 76 (1969) 791). For some time I have considered a problem which I feel is of much more fundamental importance.

We define a graph $\mathfrak{G}(M)$ as follows: the vertices of our graph $\mathfrak{G}(M)$ are the mathematicians. Two vertices are joined if the corresponding mathematicians have written at least one joint paper. (For the time being, let us ignore the papers with more than two authors.)

Is $\mathfrak{G}(M)$ planar, that is, can it be imbedded in E^2? I was not able to solve this interesting and important question. It seems that $\mathfrak{G}(M)$ does not, at present, contain a complete pentagon $\mathfrak{K}(5)$. It certainly contains a $\mathfrak{K}(4)$; for example, in the set Erdös-Rényi-Szekeres-Turan, each pair has a joint paper.

I communicated this problem to Schinzel, who proved that $\mathfrak{G}(M)$ is not planar by showing that $\mathfrak{G}(M)$ contains a $\mathfrak{K}(3,3)$ — that is, a complete bipartite graph of 6 vertices (with three vertices of each color and the 9 edges connecting black to white in all possible ways). The white vertices are Chowla, Mahler, Schinzel; the black ones are Davenport, Erdös, Lewis; the simple task of finding the 9 relevant papers can be left to the reader.

I would like to mention some interesting related problems. There are sets of three mathematicians, each subset of which has a paper (more precisely, only the empty set has no papers); for example, Erdös-Rogers-Taylor. It would be nice to have an example of a set of 4 mathematicians where each of the 15 non-empty subsets has a paper. I believe such a set does not yet exist.

The graph $\mathfrak{G}(M)$, in fact, should be denoted by $\mathfrak{G}^{(t)}(M)$ (t stands for time). I suggest the following optimistic conjecture: to each integer r there is a time t_r so that for $t > t_r$ the graph $\mathfrak{G}^{(t)}(M)$ contains a complete graph $\mathfrak{K}(r)$ of r vertices.