PUBLIKACIJE ELEKTROTEHNIČKOG FAKULTETA UNIVERZITETA U BEOGRADU
PUBLICATIONS DE LA FACULTÉ D'ÉLECTROTECHNIQUE DE L'UNIVERSITÉ À BELGRADE

SERIJA: MATEMATIKA I FIZIKA — SÉRIE: MATHÉMATIQUES ET PHYSIQUE

№ 412 — № 460 (1973)

434. A TRIANGLE INEQUALITY*

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It is a known result [1, 13.5] that if A, B, C denote the angles of a triangle, then

$$\cos^2(A/2), \quad \cos^2(B/2), \quad \cos^2(C/2)$$

are possible sides of another triangle. It then follows immediately, that

$$\cos(A/2), \quad \cos(B/2), \quad \cos(C/2)$$

are also sides of a triangle [1, 13.6]. We show, more generally, that

$$\cos^\lambda(A/\lambda), \quad \cos^\lambda(B/\lambda), \quad \cos^\lambda(C/\lambda)$$

are sides of a triangle for all real $\lambda \geqq 2$.

If $A \geqq B \geqq C$, it suffices to show that

$$\cos^\lambda(A/\lambda) + \cos^\lambda(B/\lambda) \geqq \cos^\lambda(C/\lambda).$$

Since $\max \cos(C/\lambda) = 1$ and $\min\{\cos^\lambda(A/\lambda) + \cos^\lambda(B/\lambda)\}$ occurs for $C = 0$, we need only prove that

$$\cos^\lambda(A/\lambda) + \cos^\lambda(B/\lambda) \geqq 1 \quad \text{for} \quad A + B = \pi.$$

For $\lambda = 2$, the l. h. s. reduces to 1. For larger values of λ, the inequality immediately follows from the

Lemma. $\cos^\lambda(A/\lambda)$ $(0 \leqq A \leqq \pi)$ *is a non-decreasing function of* λ *for* $\lambda \geqq 2$.

Proof. It suffices to prove that $dy/d\lambda \geqq 0$ where $y = \cos^\lambda(A/\lambda)$. Here, $y'/y = x \tan x + \log \cos x$ where $x = A/\lambda$. Then $D_x\{y'/y\} = x \sec^2 x \geqq 0$. Also, $\log y$ is concave (in λ).

Finally, corresponding to [1, 13.6)],

$$\cos^\mu(A/\lambda), \quad \cos^\mu(B/\lambda), \quad \cos^\mu(C/\lambda)$$

are sides of a triangle where $\lambda \geqq \mu \geqq 0$, $\lambda \geqq 2$.

* Presented November 24, 1972 by O. Bottema.

REFERENCES

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