and it has been conjectured that n may be replaced by $m \leq n$ where m is the length of the longest circuit (with no repeated nodes or edges) in the graph of A. We show that when A is doubly stochastic this conjecture is correct not only for the eigenvalues of A but also for all elements of the field of values of A.

Note: The conjecture mentioned above has recently been proven by R. B. Kellogg and A. B. Stephens in a paper to appear in Linear Algebra and Appl.

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On the largest prime factors of n and $n+1$

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Let $P(n)$ denote the largest prime factor of n, let $A(x, t)$ denote the number of $n \leq x$ with $P(n) \geq x^t$, and let $B(x, t, s)$ denote the number of $n \leq x$ with $P(n) \geq x^t$ and $P(n+1) \geq x^s$. A classical result of Dickman is that

$$a(t) = \lim_{x \to \infty} x^{-1} A(x, t)$$

is defined and continuous on $[0, 1]$. It is natural to guess that

$$b(t, s) = \lim_{x \to \infty} x^{-1} B(x, t, s)$$

is defined and continuous on $[0, 1]^2$ and that $b(t, s) = a(t)a(s)$. Lending some support to these guesses, we prove that for each $\epsilon > 0$, there is a $\delta > 0$ such that the number of $n \leq x$ with

$$x^{-\delta} < P(n)/P(n+1) < x^{\delta}$$

is less than ϵx. Our proof entails Brun's method. A corollary is that the probability that $P(n) > P(n+1)$ is positive (almost certainly this probability is 1/2).

Our methods allow us to say something about integers n which have the same sum of their prime factors as $n+1$. We prove the number of such $n \leq x$ is $O(x/(\log x)^{1-\epsilon})$ for every $\epsilon > 0$. We know a proof in the case $\epsilon = 0$ as well, but it is more complicated and not presented.

In addition we present a brief discussion on the largest prime factors of 3 or more consecutive numbers.

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